

MATH 1342 – STATISTICS

SECTION 8.2: A SINGLE POPULATION MEAN USING THE STUDENT t DISTRIBUTION

- In practice, we rarely know the population standard deviation.
- If you draw a simple random sample of size n from a population that has an approximately normal distribution with mean μ and unknown population standard deviation σ and calculate the t -score $t = \frac{\bar{x} - \mu}{\left(\frac{s}{\sqrt{n}}\right)}$, then the t -scores follow a Student's t -distribution with $n - 1$ degrees of freedom.

$$z = \frac{x - \mu}{\sigma}$$

The t -score has the same interpretation as the z -score. It measures how far $\bar{x}$ is from its mean μ . For each sample size n , there is a different Student's t -distribution.

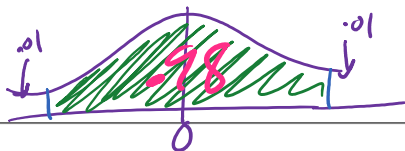
- The **degrees of freedom (df)**, $n - 1$, come from using n deviations to find s . Because the sum of the deviations is zero, we can find the last deviation once we know the other $n - 1$ deviations. The other $n - 1$ deviations can change or vary freely.
- Properties of the Student's t -Distribution
 - The graph for the Student's t -distribution is similar to the standard normal curve.
 - The mean for the Student's t -distribution is zero and the distribution is symmetric about zero.
 - The Student's t -distribution has more probability in its tails than the standard normal distribution because the spread of the t -distribution is greater than the spread of the standard normal. So the graph of the Student's t -distribution will be thicker in the tails and shorter in the center than the graph of the standard normal distribution.
 - The exact shape of the Student's t -distribution depends on the degrees of freedom. As the degrees of freedom increases, the graph of Student's t -distribution becomes more like the graph of the standard normal distribution.
 - The underlying population of individual observations is assumed to be normally distributed with unknown population mean μ and unknown population standard deviation σ . The size of the underlying population is generally not relevant unless it is very small. If it is bell-shaped (normal) then the assumption is met and doesn't need discussion. Random sampling is assumed, but that is a completely separate assumption from normality.
- With TI-83,83+, and 84+: tcdf (lower bound, upper bound, degrees of freedom).
- However for confidence intervals, we need to use inverse probability to find the value of t when we know the probability.
- The notation for the Student's t -distribution (using T as the random variable) is: $T \sim t_{df}$ where $df = n - 1$. (If $n = 20$, then $T \sim t_{19}$)
- If the population standard deviation is unknown, the **margin of error, ME** , for a population mean is:

$$ME = (t_{\alpha/2}) \left(\frac{s}{\sqrt{n}} \right)$$

- $t_{\alpha/2}$ is the t -score with area to the right equal to $\frac{\alpha}{2}$.
- use $df = n - 1$ degrees of freedom, and
- s = sample standard deviation.

- The format for the confidence interval is: $(\bar{x} - ME, \bar{x} + ME)$.





Ex. 1: Suppose that the amount of money spent per week on groceries is normally distributed with an unknown mean and standard deviation. A random sample of 24 grocery bills is taken and gives a sample mean of \$105 and a sample standard deviation of \$18. Find the ME (margin of error), for a 98% confidence interval estimate for the population mean using the Student's t -distribution. Then find the confidence interval.

(a) Using the t -table.

$$n = 24 \Rightarrow df = 23$$

$$n-1 = 24-1$$

$$\bar{x} = 105$$

$$s = 18$$

$$CL = .98$$

$$\alpha = 1 - .98 = .02$$

$$\frac{\alpha}{2} = \frac{.02}{2} = .01$$

$$t_{\frac{\alpha}{2}} = t_{.01} = 2.50$$

$$df = 23$$

$$ME = t_{.01} \left(\frac{s}{\sqrt{n}} \right)$$

$$= 2.50 \left(\frac{18}{\sqrt{24}} \right)$$

$$= 9.1855865$$

$$\text{Confidence interval: } (\bar{x} - ME, \bar{x} + ME)$$

$$(105 - 9.1855865, 105 + 9.1855865)$$

$$(95.8144, 114.1856)$$

(b) Using the calculator.

STAT $\rightarrow$ TEST
TInterval

$$ME = \bar{x} - \text{endpt}$$

$$\bar{x} = 105$$

$$s_x = 18$$

$$n = 24$$

$$CL = .98$$

$$(95.815, 114.19)$$

conf. interval

$$105 - 9.185$$

$$= 9.185$$

To Find a Confidence Interval for a Population Mean for a Student t Distribution (Page 454)

(This is assuming you have entered data in a list.)

- (1.) Press [STAT] and arrow over to [TESTS].
- (2.) Arrow down to 8:TInterval and press [ENTER]. (Or you can just press 8.)
- (3.) Arrow down to List and enter the list name where you put the data.
- (4.) There should be a 1 after Freq.
- (5.) Arrow down to C-level and enter a decimal.
- (6.) Arrow down to Calculate and press [ENTER].
- (7.) The confidence interval is displayed. Recall this is $(\bar{x} - ME, \bar{x} + ME)$.

Ex. 2: A random sample of statistics students were asked to estimate the total number of hours they spend watching television in an average week. The responses are recorded below. Use this sample data to construct a 98% confidence interval for the mean number of hours statistics students will spend watching television in one week.

L1 $\rightarrow$ 0, 3, 1, 20, 9, 5, 10, 1, 10, 4, 14, 2, 4, 4, 5

(a) Using the t -table.

1-Var stat:

$$\bar{x} = 6.13$$

$$s_x = 5.514483095$$

$$n = 15 \Rightarrow df = 14$$

$$CL = .98 \Rightarrow \alpha = .02$$

$$\frac{\alpha}{2} = .01$$

$$t_{.01} = 2.62$$

$$df = 14$$

$$ME = t_{.01} \left(\frac{s}{\sqrt{n}} \right) = 2.62 \left(\frac{5.514483095}{\sqrt{15}} \right)$$

$$\approx 3.7304$$

vars $\rightarrow$ 5: statistics
 $\rightarrow$ 3: s_x

$$(6.1333 - 3.7304, 6.1333 + 3.7304)$$

(b) Using the calculator.

STAT $\rightarrow$ TESTS

TInterval

$$(2.3965, 9.8702)$$